EXAMINATION II:

Fixed Income Valuation and Analysis

Derivatives Valuation and Analysis

Portfolio Management

Solutions

Final Examination

March 2007

Question 1: Fixed Income Valuation and Analysis **(50 points)**

a)
The price for a short-term interest rate futures contract is quoted as "100 – implied forward interest rate". Therefore, according to the pure expectations hypothesis, the interest-rate corresponding to the September 2007 contract month is 2.65%, which is a 0.65 percentage point rise from current levels; the interest-rate corresponding to the March 2008 contract month is 3.85%, which is a 1.85 percentage point rise expected by the market. If the market is demanding a positive risk premium, the expected rise in interest rates is less than these figures indicate.

P.S: Contrary to the forward contract, there is no convexity premium in the futures.

b)
Sub-decimal differences due to differences in the calculation process can be tolerated as long as the calculation logic is correct:

$$(B):\ \frac{1}{(1+X)^5} = 0.8102 \Rightarrow X = \sqrt[5]{\frac{1}{0.8102}} - 1 = 0.042993\cdots \Rightarrow 4.299\%$$

$$(C):\ X = \frac{1}{(1+0.0312)^2} = 0.9404$$

$$(D):\ \text{Since } (1+R_6)^6 \cdot (1+X) = (1+R_7)^7 \Rightarrow$$

$$\Rightarrow X = \frac{(1+R_7)^7}{(1+R_6)^6} - 1 = \frac{df_6}{df_7} - 1 = \frac{0.7701}{0.7348} - 1 = 0.048040\cdots \Rightarrow 4.804\%.$$

(A): $0.9756 \cdot X + 0.9404 \cdot X + 0.8988 \cdot (1+X) = 1$

$$\Rightarrow X = \frac{1-0.8988}{0.9756+0.9404+0.8988} = 0.035953\cdots \Rightarrow 3.595\%$$

(E) : D = $((1 \cdot 3.11/1.0311) + 2 \cdot (3.11+100) / 1.0311^2)/100 = 1.9698$ and modified duration is Dmod = 1.9698 / 1.0311 = 1.910

c)
For 1), it is the 4-year zero coupon bond, because 4 years is the maturity with the highest 1-year forward rate. In fact, if the yield curve remains unchanged, the return for a holding period will be equivalent to the corresponding forward rate, and the forward 1-year interest rate is highest beginning three years from now, for the 4-year zero-coupon bond.

Proof:

Today the price of a bond maturing in t years is: $P_{today} = \frac{100}{(1+R_t)^t}$

In 1 year the price of this same bond is: $P_{1year} = \frac{100}{(1+R_{t-1})^{t-1}}$

The return from holding this bond during one year is: $\frac{P_{1year}}{P_{today}} - 1 = \frac{(1+R_t)^t}{(1+R_{t-1})^{t-1}} - 1 = F$

For 2) the holding period return is the same for zero-coupon bonds of all maturities, and is equal to the 1-year zero-coupon bond yield, i.e. 2.5%.
The reason is that, for every maturity n (in years), we have: $(1+R_1)\cdot(1+F_{1,n})^{n-1} = (1+R_n)^n$

so the 1-year return is: $\frac{P_{1year}}{P_{today}} - 1 = \frac{(1+R_n)^n}{(1+F_{1,n})^{n-1}} - 1 = R_1$.

d)
Be X the weight of the 3-year bond. Then
2.797 · X + 5.907 · (1-X) = 4.422 ⇒ X = 0.4775 and (1-X) = 0.5225.
Therefore the market value of the barbell portfolio's 3-year par bond is 47.75 and the one of the 7-year par bond is 52.25.

e)
For case 1, the performance of the barbell portfolio will be higher if there is a large parallel shift. This is because the barbell portfolio has higher convexity and its relative duration will become shorter as yields rise.

For case 2, the performance of the bullet portfolio will be higher.

Solution A:
For the yield curve to become completely flat and equivalent to the 5-year bond yield, the 3-year bond yield must rise 0.644% and the 7-year bond yield decline 0.193%.
On the other hand, the answer to question d) has shown that the ratio of modified duration for the 3-year bond to 7-year bond in the barbell portfolio is:

$$\frac{2.797 \cdot 47.75}{5.907 \cdot 52.25} = \frac{1}{2.311}$$

Multiply both by changes in yield above to see the ratio of market price change, we obtain:

$$\frac{-0.644}{2.311 \cdot 0.193} = \frac{-0.644}{0.446}$$

So the rise in the yield for the 3-year bond has a larger impact. Therefore, returns on the barbell portfolio will be negative. On the other hand, the bullet portfolio will not see any change in its market price because the yield on the 5-year bond remains the same.

Solution B:
If the yield curve becomes flat, then the par yields are equal to the zero coupon yields (= 4.239%). Using that information you can calculate the "true" market value of the 3-year par bond (paying a coupon of 3.595) and the 7-year par bond (paying a coupon 4.432%) using a discount rate of 4.239% for each cash flow. The 3 resp. 7 year bond will have a new market value of 98.2204 resp. 101.148. Using the investment weights 47.75% / 52.25% this results in a new market value of the barbell portfolio of 99.7502 < 100 (= the bullet portfolio).

f)
The mortgage backed securities have sold an option to the borrower of the home loan that allows him to accelerate repayment. Greater volatility in market interest rates will (assuming all else is equal) cause this option to increase in value, which can be expected to lower the price of the mortgage backed security portfolio. To hedge the impact of higher interest rate volatility, since the company is short in interest rate volatility, it should purchase interest rate options such as bond options or swaptions.

Question 2: Portfolio Management **(36 points)**

a)
The “asset allocation effects” gives the contribution to the returns produced by intentional deviation in asset allocation from the fund’s policy asset mix and by deviation caused by market price changes.

The “security selection effects.” refers to the returns obtained by active management rather than passive management against the benchmark.

The asset allocation effects can be measured by subtracting the returns achieved through passive management conforming to the policy asset mix (I) from the returns obtained by active asset allocation (II):
asset allocation effects = (II) - (I)

The security selection effects can be found by deducting the returns obtained by passive management (I) from the returns obtained by active security selection (III):
security selection effects = (III) - (I)

b)

Fiscal year	Security selection effects	Asset allocation effects	Cross effects	Total active effects
1999	4.0	-2.8	-0.4	0.8
2000	0.5	-3.0	0.1	-2.4
2001	2.0	-1.5	0.2	0.7
2002	1.0	-3.0	0.0	-2.0
2003	-2.0	4.7	-0.2	2.5
2004	-8.0	0.0	0.0	-8.0
2005	**-3.0**	**5.2**	**0.2**	**2.4**

Total active effects = (IV) – (I)
$= \Sigma w_{P,j} \cdot R_{P,j} - \Sigma w_{I,j} \cdot R_{I,j} = [0.4 \cdot (-6) + 0.6 \cdot 48] - [0.5 \cdot (-2) + 0.5 \cdot 50] = 26.4 - 24 = 2.4$

Security selection effects = (III) - (I)
$= \Sigma w_{I,j} \cdot [R_{P,j} - R_{I,j}] = 0.5 \cdot [-6 - (-2)] + 0.5 \cdot [48 - 50]) = -3$

Asset allocation effects = (II) - (I)
$= \Sigma[w_{P,j} - w_{I,j}] \cdot R_{I,j} = [0.4 - 0.5] \cdot (-2) + [0.6 - 0.5] \cdot 50 = 5.2$

Cross effects = Total active effects - Security selection effects - Asset allocation effects
= 2.4 - (-3.0) - 5.2 = 0.2

Comment:
The security selection effects were positive for the 4-year period beginning 1999, but turned negative in 2003. The total security selection effect over the 7 years was -5.5%.
The asset allocation effect was negative during the first 4 years but turned positive for the 3 years beginning 2003. Its 7-year total was roughly 0%.
The total active effect including the cross effect was -6% for the 7-year period, but the -8%

issue selection effect in 2004 was a major factor in this.

c)
The beta of the equity portfolio is 0.9926. In other words, the portfolio has a sensitivity against the benchmark of nearly 1, so a 1% rise (fall) in the benchmark will produce a 1% rise (fall) in the portfolio as well.

On the other hand, the alpha is -1.651%, so active management has not been effective.

The coefficient of determination of 0.9454 indicates that 94.54% of the fluctuation in portfolio returns can be explained by fluctuation in benchmark returns.

d)
In the bond portfolio, Issue A is a short-term corporate bond and the fund is taking credit risk, while Issue B is a long-term government bond and the fund is taking interest-rate risk. Issue B has a higher weight within the portfolio, resulting in a portfolio duration of
$20\% \cdot 2.9 + 80\% \cdot 11.8 = 10$ years

The benchmark has a duration of 5 years, so the portfolio is more sensitive to interest rates than the benchmark. For example, a 1% rise in interest rates would result in approximatively 10% decline in portfolio pricing [exact: 10%/(1+YTM)] instead of approx. 5% decline in benchmark.

e)
Rising interest rates result in falling bond prices and avoiding this will require to reduce the weighting of bonds in the portfolio from the current 40%. In addition, Issue B, which is a long-term bond, could be sold and replaced with bonds of shorter duration so as to reduce the duration of the portfolio as a whole and mitigate interest-rate risk.

Question 3: Portfolio Management (21 points)

a)

- Time horizon: His time horizon is 16 years. This is a long period, which could allow to take more risk than for a short term time horizon.
- Liquidity needs: He does not need much liquidity until his retirement.
- Risk tolerance: He seems to be modestly risk-averse.
- Tax: He seems to be in a higher tax bracket.
- Specific needs: To sustain consumption or living standard after retirement, he has to preserve asset values in real terms.

b)

Dr. K's objective is to grow from USD 400,000 to USD 1 million over 16 years.

The required rate of return for this goal is calculated as follows:

$$r=\frac{1}{16}\cdot \ln\left(\frac{1'000'000}{400'000}\right)=5.73\%$$

(in simple terms: $\sqrt[16]{\frac{1,000,000}{400,000}}-1=1.0589=5.9\%$)

c)

Be σ_T is the standard deviation of the return for T years. Then $\sigma_T=\sqrt{T}\cdot\sigma$, with T=16.

If the actual return over the next 16 years turns out to be one standard deviation below the expected return of the portfolio he selects, he will obtain:

$16\cdot R-\sigma\cdot\sqrt{16}$, which per annum corresponds to $R-\frac{\sigma}{\sqrt{16}}$.

Portfolio number	1	2	3	4	5	6
Exp. Return	5.0	5.7	6.3	7.0	7.7	8.4
Std. Dev. σ	7.1	7.0	8.1	9.9	12.2	14.7
Std. Dev. $\frac{\sigma}{\sqrt{16}}$	1.78	1.75	2.03	2.48	3.05	3.68
1 Std. Dev. below expected return	3.23	3.95	4.28	4.53	4.65	4.73
Value at 16th year	670.13	752.55	792.72	825.07	841.73	851.90
Note for the correctors: The above calculation is based on continuously compounding, however, calculating by simple terms is also acceptable as shown below. Your conclusion must be the same.						
Value at 16th year	664.69	743.45	781.53	812.05	827.73	837.27

To provide downside protection for him, portfolios 4, 5 and 6 are available choices.

Since Dr. K seems risk averse, portfolio 4 could be appropriate, since it is the least volatile one. But also portfolio 6 could be a good choice, since the Worst Minimum Wealth (defined as in the text) accepted by Dr. K is the highest for Portfolio 6.

He has to take inflation into consideration. Future asset value should be preserved in real terms after adjusted for inflation. For example, if inflation is running at 5% per year, Portfolio 1 does not earn extra return over inflation, and his asset value in real terms will remain at 400,000 after 16 years.

Question 4: Derivatives and Portfolio Management (73 points)

a)
To end up with EUR 10 million we have to invest EUR
$\frac{10{,}000{,}000}{1.00675} = 9{,}932{,}952.57$ for three month. This is equivalent to USD
$9{,}932{,}952.57 \cdot 1.1980 = 11{,}899{,}677.18$ converted at the current spot rate. This amount borrowed today costs USD $11{,}899{,}677.18 \cdot 1.012 = 12{,}042{,}473.31$ after three month.
Therefore the total amount paid for 10 million Euros repayment is USD $12{,}042{,}473.31$.

(this in fact corresponds to the forward rate of $1.1980 \cdot \frac{1+\frac{4.8\%}{4}}{1+\frac{2.7\%}{4}} = 1.2042$).

b)
The company is short [=will have a liability of] EUR 10 million three months from now. 10 million / 62,500 = 160. Following the put-call-parity, we purchase 160 calls and sell 160 puts, both with strike 1.20. This costs USD 42,504.00:

	Amount in USD
Buy 160 Calls	160 · 62,500 · 0.02113 = -211,300.00
Sell 160 Puts	160 · 62,500 · 0.01693 = 169,300.00
Total today	- 42,000.00
Value at Maturity	-42,000.00 · 1.012 = -42,504.00

After three month, we pay in any way USD $160 \cdot 62{,}500 \cdot 1.20 = 12{,}000{,}000$ for the company's obligation. The total amount paid for 10 million Euros repayment is therefore USD 12,042,504. The small difference to a) is due to rounding effects caused by quotation conventions of the option prices.

c)
To hedge the exposure of EUR 10 million short [i.e the future liability of EUR 10 million], we need 10 million / 125,000 = 80 CME Euro FX Futures contracts long.
At delivery we have to pay USD. This is about USD 20,504[= 12,042,504 - 12,022,000] cheaper than the hedge with options.

Due to the marking to market procedure a considerable amount of liquidity can become necessary to maintain the futures hedge. This depends on the path of the spot rate over time.

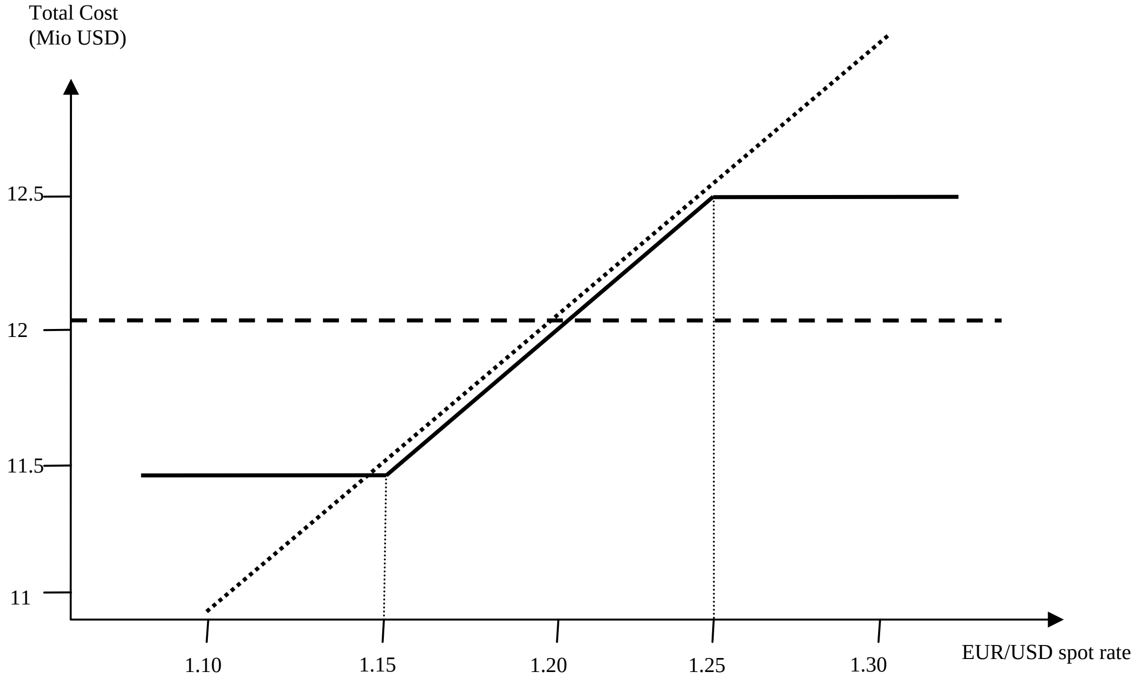
d)
We sell 10 million / 62,500 = 160 puts with strike 1.15 and purchase 160 calls with strike 1.25.

This results in an income of 10 million · (0.00416 - 0.00312) = 10 million · (0.00104) = USD 10,400, which at maturity become USD 10,400 · 1.012 = 10,524.80

If the spot rate after three months is USD 1.30, exercising the call we buy the EUR at 1.25 and our total costs is 12,500,000 – 10,524.80 = USD 12,489,475.2.

If the spot rate after three months is USD 1.10, the put is exercised and we are forced to buy the EUR at 1.15. Hence our total costs are 11,500,000 – 10,524.80 = USD 11,489,475.2.

The graph shows the total costs of the 10 million EUR obligation (bold line) calculated at maturity of the options compared with the total hedge from c) (dotted line) and the unhedged position (pointed line).



e)
To replicate the strategy, we need a futures position with the same position delta as the option position. That is

$$N_F \cdot \Delta_F \cdot k_F = N_C \cdot \Delta_C \cdot k_C - N_P \cdot \Delta_P \cdot k_P .$$

With $\Delta_C = 0.1464$ and $\Delta_P = -0.1197$ and a futures delta of $\Delta_F = \frac{1.012}{1.00675} = 1.0052148$,

we get:

$$N_F = \frac{N_C \cdot \Delta_C \cdot k_C - N_P \cdot \Delta_P \cdot k_P}{\Delta_F \cdot k_F}$$

$$= \frac{60 \cdot (0.1464) \cdot 62'500 - 60 \cdot (-0.1197) \cdot 62'500}{(1.0052148) \cdot 125'000} = 7.942$$

Therefore we have to buy 8 CME Euro FX Futures to start the replication.

f)
Frequent problems are:
- it requires (in theory) continuous trading;
- it cannot take jumps into consideration;
- it requires a constant volatility;
- interest rates should be both borrowing and lending rates;
- it depends on very low transaction costs.

g)
The Put-call parity with European options on assets paying known dividends yield y is: $C_E - P_E - S \cdot e^{-y \cdot t} + Ke^{-r \cdot t} = 0$. A foreign currency (here: the EUR) can be regarded as an investment asset paying a known dividend, namely the risk free rate of interest in the foreign currency. In this exercise simple rates are given, so the formula has to be correspondingly changed.

With
R_{USD} = 4.8% , R_{EUR} = 2.7% , T = 0.25 we get:

$$C_E - P_E - \frac{S}{1 + R_{EUR} \cdot T} + \frac{K}{1 + R_{USD} \cdot T} = 0 \Leftrightarrow C_E - P_E - \frac{1.1980}{1.00675} + \frac{K}{1.012} = 0$$

[cf Hull, Fundamentals of futures and option markets, 4th edition, chap.12.5]

h)
h1)
1.20 and 1.25 show the same volatility for put and call but not for 1.15. So if all the option prices are right, there must be no arbitrage for 1.20 and 1.25.

h2)

Strike K [USD]	Call (USD-cent per 1 EUR)	Put (USD-cent per 1 EUR)	$C_E - P_E - \frac{1.1980}{1.00675} + \frac{K}{1.012}$
1.15	6.247	0.416	0.4706
1.20	2.113	1.693	0.0003
1.25	0.312	4.833	0.0000

Compared with the 1.15 put, the 1.15 call is overvalued, whereas for the 1.25 options there is no arbitrage opportunity.

Therefore the following arbitrage results in a profitable riskless position: we sell the real call and we buy the synthetic call created through: long put EUR/USD, long underlying (i.e. long the discounted value of 1 EUR) and short the discounted value of the strike (i.e. short the discounted value of 1.15 USD).

	USD per 1 EUR	USD per PHLX option
Sell Call	0.06247	3904.38
Buy Put	-0. 00416	-260.00
Buy Underlying (discounted value of 1 Eur. Here is reported its countervalue in Usd)	-(1.1980/1.00675) = -1.18997	-74,372.98
Borrow the discounted value of the strike, i.e. of 1.15 USD	1.13636	71,022.73
Total	0.004706	294.12

At maturity, *if the EUR/USD is below 1.15*, the call is not exercised. We are long 1 Eur, which we sell using the put at 1.15, getting 1.15 USD which are used to refund the debt of 1.15 USD. *If* at the contrary *at maturity the EUR/USD is above 1.15*, the call is exercised: the 1 Eur is sold for 1.15 USd, which are then used to refund the borrowing of 1.15 USD. The put is not exercised.

In any case we end up with a flat position. And at the beginning we have cashed-in 294.12 USD per PHLX option.